Investigating into the novel solitary wave and lump wave solution in fluid Dynamics

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Abstract:

We used the unified method in this study to precisely solve the well-known the time fractional Drinfel'd-Sokolov-Wilson equation. The equation's graphic solution depicts a variety of physical phenomena, including periodic soliton behavior, a lump wave, a bright kink, a singular kink, a kink with interaction, and a kink with interaction. NLPDEs in mathematical physics and all other engineering fields can be solved using this method. It is a mathematical tool that is extremely potent and efficient.

Keywords: The time fractional Drinfel'd-Sokolov–Wilson equation, The fractional derivative, The Unified Method, Dispersive Water Waves.

1. Introduction:

The study of exact solutions of NLPDEs plays a vital role in the inquisition of nonlinear physical phenomena like as fluid dynamics, visual strands, electrical conduction shapes, plasma physical science, artificial intelligence, engineering, physics, earth sciences, and bioinformatics and so on. In short, it is a fundamental ingredient of all modern sciences. In future, it may be focused on fluid mechanics, optimal control and biochemical problems. At present, various effective methods for gaining exact solution of NLPDEs have been introduced, such as Bäcklund transformation method [3], Homogeneous balance method [1,2], Bifurcation method [6,7,8], The hyperbolic tangent function expansion method [4,5], The Jacobi elliptic function method [9,10,12], Hirotas bilinear method[11] and so on.

In this paper, we consider the classical Drinfel'd-Sokolov–Wilson equation, [13]

$$\begin{aligned} m_t + l_1 n n_x &= 0, \\ n_t + l_2 n_{xxx} + l_3 m n_x + l_4 m_x n &= 0. \end{aligned} \tag{1}$$

Where l_1, l_2, l_3, l_4 are some nonzero parameters.

At present, Drinfel'd-Sokolov–Wilson equation and the coupled Drinfel'd-Sokolov–Wilson equation, a special case of Drinfel'd-Sokolov–Wilson equation have been investigated by many authors [14-20]. In this investigation, we use complex variation ξ defined as $\xi = Hx - Ct$, now we transfer the equation (1) in ODE which write,

$$-Cm' + l_1 n H n' = 0 \tag{2}$$

$$-Cn' + l_2 H^3 n''' + l_3 H m n' + l_4 H n m' = 0 \tag{3}$$

Where the das or prime denotes the differentiation with the help of ξ. Integrating (2) we get,

$$m = \frac{l_1 H n^2}{2C} + h$$

Where h is an integrating constant.

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And then

$$m' = \frac{l_1 Hnn'}{C}$$

Putting the value of m and m' in equation (3) we obtain,

$$2Cl_2H^3n''' + (2Cl_3hH - 2C^2)n' + (l_3l_1H^2 + 2l_1l_4H^2)n^2n' = 0 \tag{4}$$

We divide this article into five sections. The introduction section is our first section. We have talked about the depiction of the strategy in the 2nd section. The 3rd section we have applied the technique in Drinfel'd-Sokolov–Wilson equation or application. The 4th section has delivered the graphical representation and discussion section. Finally, we have given our conclusion in the 5th section.

2. Enlargement of the Unified Method:

In this portion, we will narrate the unified method for determining different types of traveling wave solutions of nonlinear evolution equations. Let us consider the NLPDEs in two independent variables x and t, is given by

$$R(m, m_x, m_t, m_{xx}, m_{tt}, m_{xt}, \ldots\ldots\ldots) = 0, \tag{5}$$

Using wave transformation:

$$m(x,t) = M(\xi), \xi = Hx - \frac{1}{\sigma}Ct^{\sigma}. \tag{6}$$

Renewed Eq. (5) to ordinary differential equation (ODE):

$$Z(M, M', M'', M''', \ldots\ldots\ldots) = 0. \tag{7}$$

Suppose the trail solution of ODE Eq. (7) is the following form:

$$M(\xi) = \alpha_0 + \sum_{i=1}^{w}\left[\alpha_i F(\xi)^i + \beta_i F(\xi)^{-i}\right], \tag{8}$$

Where α_i $(i = 0, 1, 2, \ldots, w)$ and $\beta_i (i = 0, 1, 2, \ldots, w)$ are constants to be investigated afterward such that α_w and β_w cannot be zero at a time. Let us consider an ODE namely Riccati differential equation:

$$F' = \left(F(\xi)\right)^2 + K. \tag{9}$$

It is satisfied by $m(\xi)$. The solution of the considering Riccati differential equation is given below:

Case-01: Hyperbolic function solutions (When $K < 0$):

$$F(\xi)=\begin{cases}\frac{\sqrt{-(\alpha^2+\beta^2)K}-\alpha\sqrt{-K}\cosh\left(2\sqrt{-K}(\xi+D)\right)}{\alpha\sinh\left(2\sqrt{-K}(\xi+D)\right)+\beta}, \\ \frac{-\sqrt{-(\alpha^2+\beta^2)K}-\alpha\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+D)\right)}{\alpha\sinh\left(2\sqrt{-\beta}(\xi+D)\right)+\beta}, \\ \sqrt{-K}+\frac{-2\alpha\sqrt{-\beta}}{\alpha+\cosh\left(2\sqrt{-K}(\xi+D)\right)-\sinh\left(2\sqrt{-K}(\xi+D)\right)}, \\ -\sqrt{-K}+\frac{2\alpha\sqrt{-K}}{\alpha+\cosh\left(2\sqrt{-K}(\xi+D)\right)+\sinh\left(2\sqrt{-K}(\xi+D)\right)},\end{cases} \tag{10}$$

Where α and β are two real arbitrary constants, and D arbitrary constant.

Case-02: Trigonometric function solutions (When $K>0$):

$$F(\xi)=\begin{cases}\frac{\sqrt{(\alpha^2-\beta^2)K}-\alpha\sqrt{K}\cos\left(2\sqrt{K}(\xi+D)\right)}{\alpha\sin\left(2\sqrt{K}(\xi+D)\right)+\beta}, \\ \frac{-\sqrt{(\alpha^2-\beta^2)K}-\alpha\sqrt{\beta}\cos\left(2\sqrt{K}(\xi+D)\right)}{\alpha\sin\left(2\sqrt{K}(\xi+D)\right)+\beta}, \\ i\sqrt{K}+\frac{-2\alpha i\sqrt{K}}{\alpha+\cos\left(2\sqrt{K}(\xi+D)\right)-i\sin\left(2\sqrt{K}(\xi+D)\right)}, \\ -i\sqrt{K}+\frac{2\alpha i\sqrt{K}}{\alpha+\cos\left(2K(\xi+D)\right)+i\sin\left(2\sqrt{K}(\xi+D)\right)},\end{cases} \tag{11}$$

Where α and β are two real arbitrary constants, and D arbitrary constant.

Case-03: Rational function solutions. (When $K=0$):

$$F(\xi)=-\frac{1}{\xi+D}, \tag{12}$$

Where $\alpha\neq 0,\beta$ and D are real arbitrary constants.

We determine the positive integer N in Eq. (7) by taking into account the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (7). Moreover, the degree of M as $G(M(\xi))=N$ which gives the order of others expression as follows:

$$G\left(\frac{d^QM}{d\xi^Q}\right)=N+Q, G\left(M^P\left(\frac{d^QM}{d\xi^Q}\right)^S\right)=PN+S(N+Q) \tag{13}$$

Inserting Eq.(8) into Eq. (7) and making use of Eq. (9) and then extracting all terms of like powers of $F(\xi)$ together, then set each coefficient of them to zero yield a over-determined system of algebraic equations and then solving this system of algebraic equations α_i $(i=0,1,2,\dots,N)$, $\beta_i(i=0,1,2,\dots,N), H$ and C. we obtain several sets of solutions.

Finally, substituting $A_i(i=0,1,2,\dots,N)$, $B_i(i=0,1,2,\dots,N)$, k and ω into Eq. (8) and using the trail solutions of Eq. (9), explicit solutions of Eq. (6) can be obtained immediately depending on the value b.

3. Application of the Unified Method:

In this paragraph we apply the unified method for equation (4) and since here the nonlinear term is n^2 and the highest order derivative is n'''. So the balance number is N=1. So the solution of equation (4) takes the following form

$$n(\xi)=\alpha_0+\alpha_1F(\xi)+\beta_1\frac{1}{F(\xi)} \tag{14}$$

Differentiating (14) with respect to ξ and putting the values of n and $n'and$ n ‴in equation (4) and equating the coefficient of $F(\xi)^i$ equal to zero (where $i = 0, \pm1, \pm2.....$). Solving those systems of equations, we obtain the solutions set for the equation (4) is:

Set1:

$$H = H, C = \frac{1}{3}\frac{l_1(l_3+2l_4)(24KH^2{l_2}^2+3hl_2l_3)H}{(l_1l_3+2l_1l_4)l_2}, \alpha_0 = 0, \alpha_1 = \pm2\sqrt{-\frac{24KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}H, \beta_1 = \mp2\sqrt{-\frac{24KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}HK$$

Set 2:

$$H = H, C = \frac{1}{3}\frac{l_1(l_3+2l_4)(-12KH^2{l_2}^2+3hl_2l_3)H}{(l_1l_2+2l_1l_4)l_2}, \alpha_0 = 0, \beta_1 = \pm2\sqrt{-\frac{-12KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}H, \beta_1 = \pm2\sqrt{-\frac{-12KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}HK$$

Set 3:

$$H = H, C = \frac{1}{3}\frac{l_1(l_3+2l_4)(6KH^2{l_2}^2+3hl_2l_3)H}{(l_1l_2+2l_1l_4)l_2}, \alpha_0 = 0, \alpha_1 = \pm2\sqrt{-\frac{6KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}H, \beta_1 = 0$$

Set 4:

$$H = H, C = \frac{1}{3}\frac{l_1(l_3+2l_4)(6KH^2{l_2}^2+3hl_2l_3)H}{(l_1l_2+2l_1l_4)l_2}, \alpha_0 = 0, \alpha_1 = 0, \beta_1 = \pm2\sqrt{-\frac{6KH^2{l_2}^2+3hl_2l_3}{l_1l_3+2l_1l_4}}HK$$

By using the unified method with above 4 set the Drinfel'd-Sokolov–Wilson equation affords exact traveling wave solutions. The solutions $n_{1,2}(x,t), n_{3,4}(x,t), n_{5,6}(x,t), n_{7,8}(x,t)$, $n_{9,10}(x,t), n_{11,12}(x,t), n_{13,14}(x,t), n_{15,16}(x,t), n_{17,18}(x,t), n_{19,20}(x,t), n_{21,22}(x,t), n_{23,24}(x,t)$, $n_{25,26}(x,t), n_{27,28}(x,t), n_{29,30}(x,t), n_{31,32}(x,t)$ all are trigonometric hyperbolic function solutions. $n_{33,34}(x,t), n_{35,36}(x,t), n_{37,38}(x,t), n_{39,40}(x,t), n_{41,42}(x,t), n_{43,44}(x,t), n_{45,46}(x,t)$ $n_{47,48}(x,t) n_{49,50}(x,t), n_{51,52}(x,t), n_{53,54}(x,t), n_{55,56}(x,t), n_{57,58}(x,t), n_{59,60}(x,t), n_{61,62}(x,t)$, and $n_{63,64}(x,t)$ all are trigonometric function solutions. And rational function result is $n_{65,66}(x,t), n_{67,68}(x,t), n_{69,70}(x,t)$.

4. Results & Discussion:

In this portion, we will deliberate the physical explanation and graphical demonstration of the gained exact and solitary wave result of the time fractional Drinfel'd-Sokolov–Wilson equation.

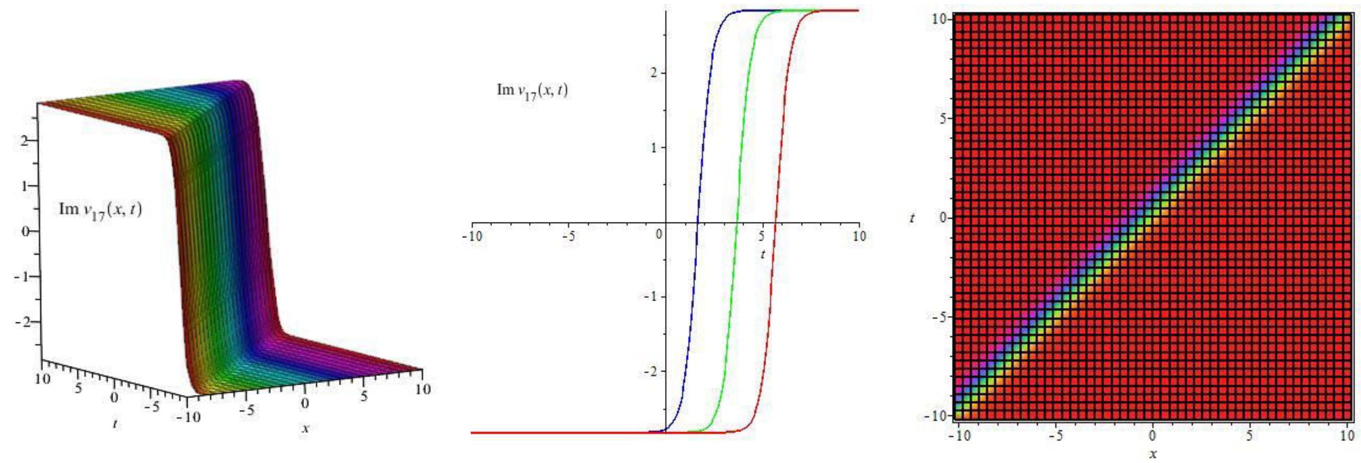
4.1 Physical Explanation:

In this sub unit, by using the unified method the time fractional Drinfel'd-Sokolov–Wilson equation affords exact traveling wave solutions. The explanation of $n_{17}(x,t)$ is a complex form and the figure shows in imaginary form which represents in Fig. 1. It looks the Dark Kink-shape type exact traveling wave solution with $k = -2, l_1 = 1, H = 1, l_2 = 1, h = 5, l_3 = 1, l_4 = 1, \alpha = 1, \beta = 1, D = 1$ within the displacements $-10 \le x \le 10$ and $-10 \le t \le 10$. So the clarification of $n_{29}(x,t)$ is a complex form and the figure shows in imaginary form which symbolizes in Fig.

2. It forms the Kink with interaction-shape type exact traveling wave solution with $K=-2, l_1=1, H=1, l_2=1, h=5, l_3=1, l_4=1, \alpha=1, \beta=1, D=1$ within the displacements $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$. And the solution of $n_{67}(x,t)$ is a complex form and the figure confirmations in real form which denotes in Fig. 3. It forms the Lump wave-shape type exact traveling wave solution with $K=0, l_1=2\sqrt{-3}, H=1, l_2=1, h=0.5, l_3=\sqrt{-9}, l_4=1, \alpha=1, \beta=1, D=1$ within the displacements $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$. Finally the solution of $n_{69}(x,t)$ is a complex form and the figure shows in imaginary form which denotes in Fig. 4. It presences the Lump wave-shape type exact traveling wave solution with $K=0, l_1=\sqrt{-3}, H=1, l_2=1, h=0.5, l_3=\sqrt{-9}, l_4=1, \alpha=1, \beta=1, D=1$ within the displacements $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$. All figure are displayed for time fractional order $\sigma = 0.1, 0.5, and\ 0.9$.

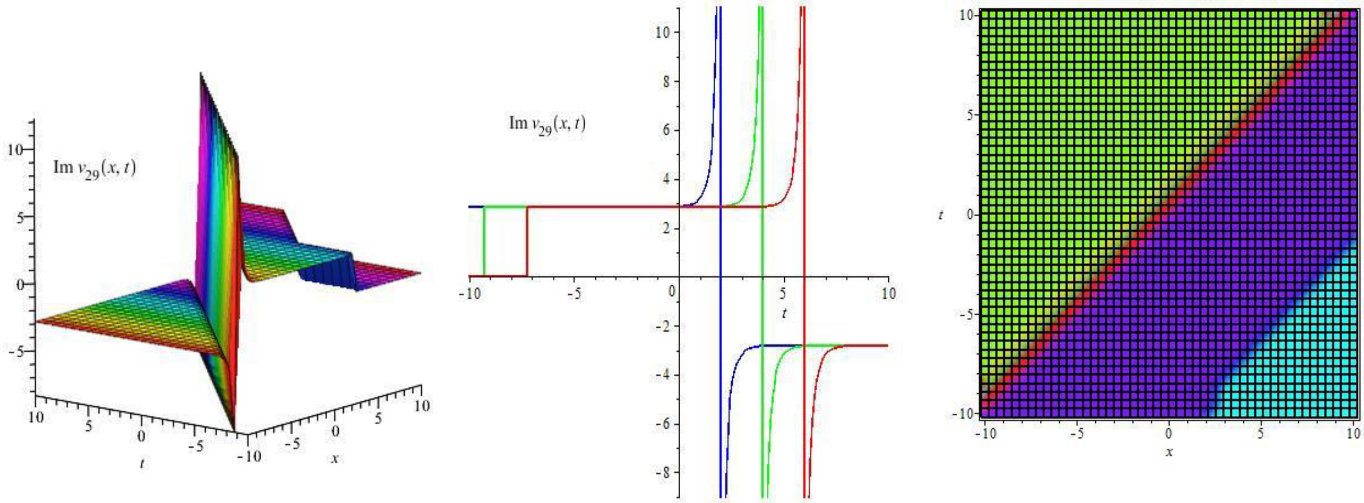
4.2 Graphically Explanation:

In this segment, we will discuss the achieved solutions by graphically with 3D, 2D and density plots. For the diverse condition on the parameters, the solutions are expressed as complex and real valued function form.



(a) (b) (c)

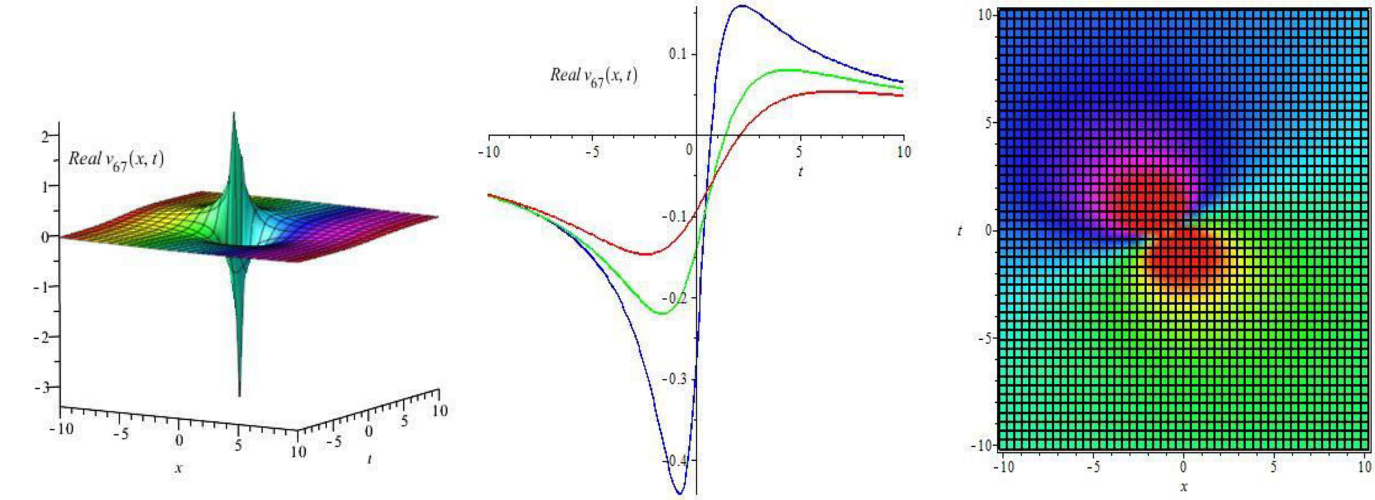
Figure 1: Structure of $n_{17}(x,t)$ for imaginary form.The figure (a) in 3D form, (b) in 2D form (c) Density plot for $t=1$.



(a) (b) (c)

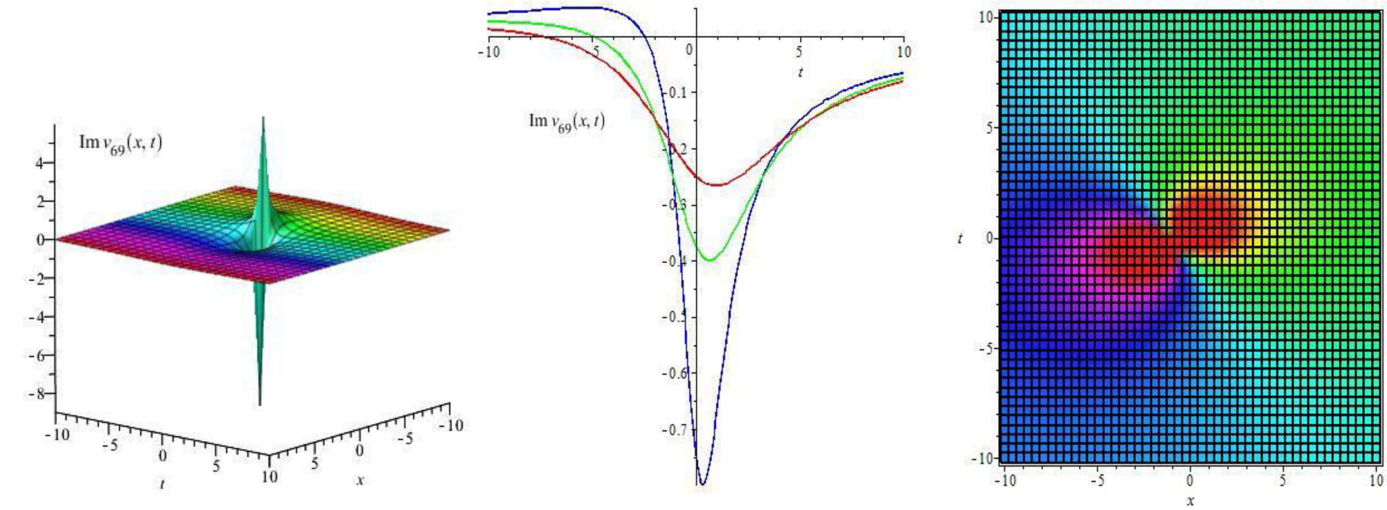
Figure 2: Structure of $n_{29}(x,t)$ for imaginary form.The figure (a) in 3D form, (b) in 2D form (c) Density

plot for $t = 1$.



(a) (b) (c)

Figure 3: Structure of $n_{67}(x,t)$ for real form.The figure (a) in 3D form, (b) in 2D form (c) Density plot for $t = 1$.



(a) (b) (c)

Figure 4: Structure of $n_{69}(x,t)$ for imaginary form.The figure (a) in 3D form, (b) in 2D form (c) Density plot for $t = 1$.

5. Conclusion:

In this article, we applied the brought together strategy effectively and this technique gives some new accurate voyaging wave arrangements of different numerical and actual charts for a few free boundaries. From this equation, we can derive particular types of displays for particular parameters. Last but not least, we believe that this approach is a powerful and easy-to-understand mathematical tool for obtaining precise traveling wave solutions. It can also be used to solve other kinds of nonlinear equations or problems.

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